FAILURE ANALYSIS CASE STUDIES II

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BREAKUP OF THE FIREWALL BETWEEN THE B AND C MODULES OF THE PIPER ALPHA PLATFORM—I. ANALYSIS BY HAND CALCULATION

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Abstract—In the Piper Alpha disaster, the initial explosion in module C destroyed the firewall between modules C and B, and fragments caused secondary damage which lead to a fire. This paper describes the dynamic analysis of the explosion response of the firewall which was not intended to resist blast and was relatively lightly constructed. The analysis is based on hand calculation of idealised elastic and elastic–plastic models. © 1998 Elsevier Science Ltd. All rights reserved.

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1. INTRODUCTION

An explosion occurred on the Piper Alpha production platform in the North Sea on 6 July 1988. In the subsequent fire 167 lives were lost, and the platform was destroyed. The incident was the subject of a major public inquiry and extensive investigations and litigation. The inquiry concluded that the initial explosion occurred in module C, and resulted from a flammable cloud created from a hydrocarbon release, at a blind flange fitted where a pressure safety valve had been removed for maintenance.

Observers saw a fireball emanate from module B at the west face of the platform some 10 s after the initial explosion. The appearance of the fireball was consistent with a condensate release. The inquiry concluded that the B/C firewall, between the C module and the B module, had been at least partially destroyed by the explosion, and that fragments from the firewall had been projected into module B and had ruptured a 4 in. pipeline carrying condensate.

The accident had unusual features. No wreckage could be examined, because most of the platform collapsed into the sea during the fire, and only the two accommodation modules were recovered. It was decided that any attempt to recover additional wreckage would not have a forensic value that would justify the risks and costs that would be incurred, since it would no longer be practicable to distinguish damage that had occurred during the original explosion from damage during the fire, collapse and subsequent recovery. On the other hand, some revealing photographs were taken less than half a minute after the explosion.

This paper describes additional research into possible explosion damage to the firewall, carried out as part of subsequent legal proceedings. The analysis is complicated by the fact that the firewall was not intended to resist pressure, and that it contains elements of widely different strengths, with a limited capability to redistribute loads.

Complex structures of this kind are nowadays almost invariably analysed by finite-element methods. However, for legal reasons that regrettably cannot be examined here, there were two independent analyses, one by hand calculations alone and the other by the finite-element method. The structure is too complicated for analysis by hand calculation to give a complete picture, but a hand calculation does give useful results which will answer the key questions that are important to an understanding of what happened. In particular, it can tell us:

1. whether or not the wall would break up;
2. when it would break up, in relation to the timing of the explosion;
3. if it broke up, how fast the fragments would be moving.

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This paper is concerned with the first and second questions, and describes the hand calculations. A second paper will describe the finite-element calculations and compare the results of the two analyses.

2. STRUCTURE OF THE B/C FIREWALL

The platform topsides between the 87 ft and 107 ft levels had four modules, identified by letters A to D from south to north. The firewalls between the modules were intended as fire barriers, and were not designed to resist blast. The B/C and A/B firewalls were similar in construction, but the C/D firewall was thicker and more massive: the significance of this point is discussed below.

The B/C firewall was 6.35 m high, and extended across the whole 46.63 m breadth of the platform. Its construction is illustrated in Fig. 1, which shows part of the wall, and details of the cross-section and clamping.

The wall was composed of rectangular and square panels within rectangular frames, some 2438 by 1524 mm (8 by 5 ft "large frames"), others 1524 by 1457 mm ("small frames"), and a few smaller. Each panel was bounded by a rectangular frame made of 50.8 × 50.8 × 6.35 mm (2 × 2 × 1/4 in.) steel angle, welded at mitre joints at the corners. The frame held a sheet of fire resistant composite, an 8.5 mm asbestos–cement core faced on both sides by 0.5 mm galvanised steel sheets, pierced by 6.35 mm tined holes on a 17.5 mm square grid. The composite sheet was bolted into the frames by 9.53 mm (3/8 in.) Whitworth bolts ("panel bolts").

The frames were bolted together by 9.53 mm bolts ("frame bolts"). There is conflicting evidence about the spacing of the frame bolts, which gave values between 15 and 24 in., and the subsequent analysis is based on an intermediate spacing of 18 in. (457.2 mm). Generally, but not always, the lowest panel was large, the next panel small, and the top panel large.

At the foot of the wall the frames were welded to the deck plate. At the top the frames were bolted to 76 × 51 mm angle cleats, which were bolted to 102 × 63 mm cleats, which were welded to the ceiling plate.

The firewall lay on the south side of the heavy trusswork which formed part of the module structural framing. The truss was a sequence of N-shaped panels. The wall was clamped to the truss by two lines of clamps, one just over 2438 mm from the floor (just above the lower bolted join between frames), and the other just over 3962 mm from the floor. Each clamp held the wall against the truss by two lengths of 9.53 mm studding. An explosion in module C would push the wall to the south, away from the trusswork, and would put the studding in tension.

There were minor variations in construction along the length of the wall, which included a door, pipe penetrations of a few composite panels, and some smaller frames.

3. OVERPRESSURE LOADING OF B/C FIREWALL BY AN EXPLOSION IN MODULE C

The inquiry report [1] examined the evidence for the size of the flammable cloud before the first explosion. It concluded that the fuel involved was condensate, and that the mass of fuel within the flammable part of the cloud was probably in the range 30–80 kg [1], Section 5.

Computational fluid dynamics (CFD) calculations were carried out by CMI [2]. The calculations are based on a gas cloud containing 45 kg of condensate, homogeneous and at stoichiometric concentration, and ignited close to the south wall. Figure 2 gives the calculated pressure history at point P1, which is in module C, on the B/C wall near the west end. The maximum pressure reached at P1 was 19.5 kN/m^2 (0.195 bar). The maximum pressures at other points within module C were higher. The CFD calculation assumed that the wall broke up when the instantaneous pressure reached 10 kN/m^2.

In the subsequent dynamic analysis the pressure history is idealised as an asymmetric triangular pulse with peak 19.5 kN/m^2, rise time 81 ms and fall time 46 ms. Figure 2 compares this idealisation with the calculated history.

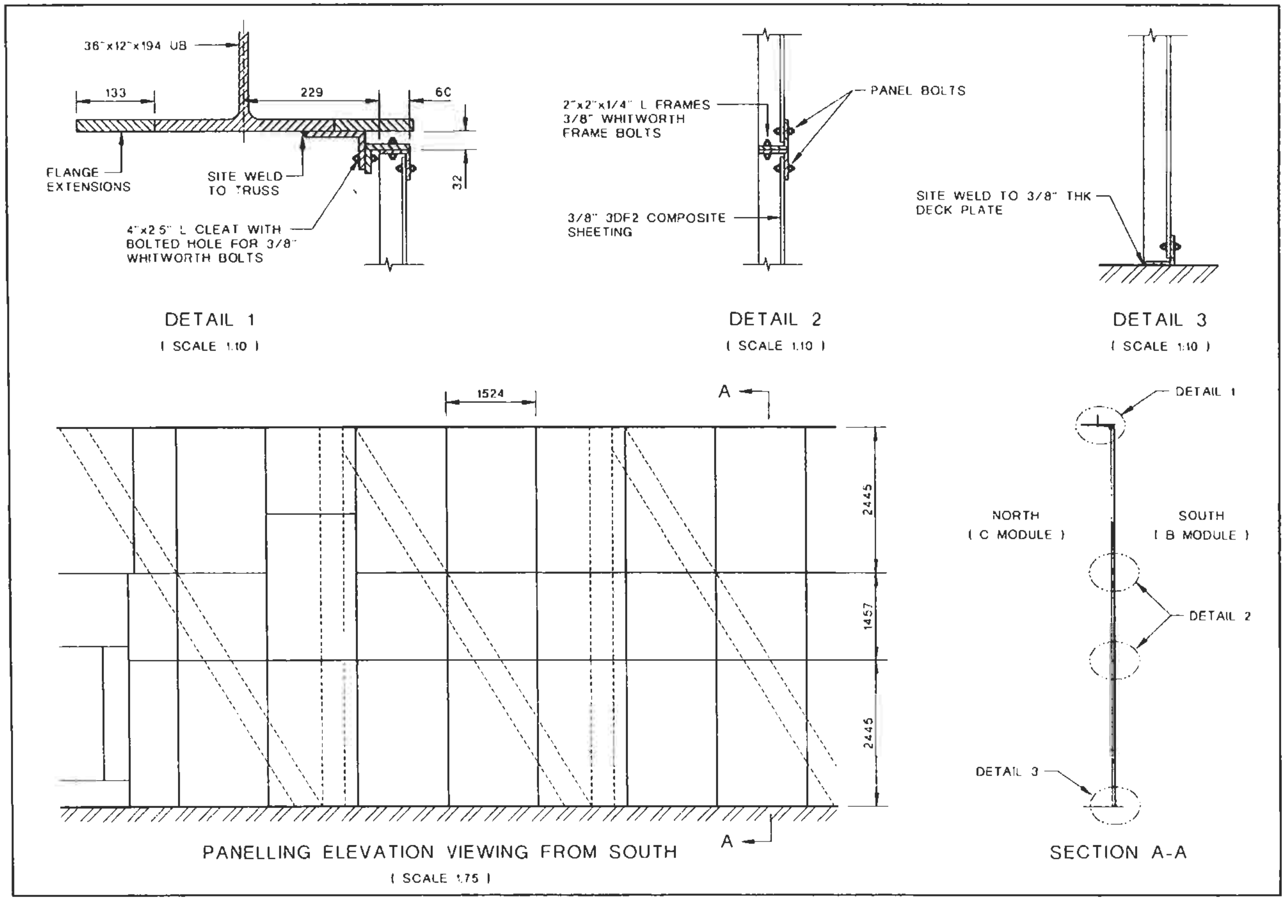

Fig. 1. B/C firewall: elevation and details.

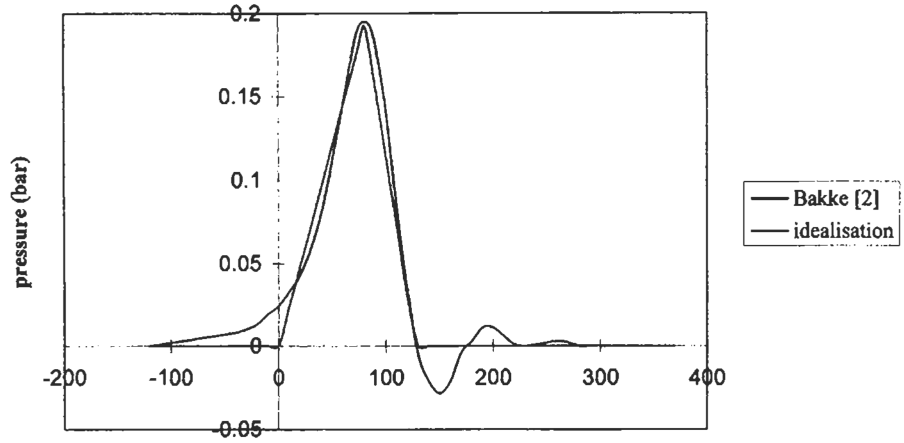

Fig. 2. Pressure pulse at point P1.

4. APPROACH TO STRUCTURAL ANALYSIS

There are several possible modes of failure of such a firewall under lateral pressure loading by an explosion. The panels can come apart by fracture of the frame bolts between them, the composite sheets can fail in bending or in tension, the panel bolts can fracture in tension or tear through the fixing holes, the angle section can fail in flexure or in torsion, and so on.

If an increasing pressure is applied to the wall, then the wall will fail in bending. Other modes of failure are possible in principle, but simple calculations show that they do not govern: in other words the wall fails in bending at a lower pressure than it would fail in other modes. For example, elementary calculations show that failure by a panel bolt hole mode, in which the panel bolts tear through the holes in the composite panels, requires a significantly higher load than several bending modes do. That is confirmed by the results of a pressure test on a small 900 mm square section of Durasteel panel, which withstood 70 kN/m^2.

Supplementary comparative calculations were carried out to check the resistance of the wall to other modes of failure, and this aspect is examined in Section 10.

5. LOCAL STRENGTH OF FIREWALL

If the dominant mode of failure is bending, the stress parameter that determines the local structural response is the bending moment stress resultant, denoted m. The limiting maximum value that m can take depends on how the panel fails locally. It can be calculated by routine methods of structural mechanics. Table 1 lists the different possible modes, gives the limited value of the moment stress resultant for each mode, and includes a simple sketch of each mode.

The local strength against frame bolt failure in tension depends on the direction of bending. If the angles connected by the bolts are loaded so that they put the bolts in tension by relative rotation around the angle toes, the maximum tension, T, in each bolt has the lever arm g_2 between the bolt axis and the toe. The bending moment per bolt is then Tg_2, and if the frame bolt spacing is s the moment stress resultant is Tg_2/s. If the direction of bending induces relative rotation about the angle heels, the lever arm is g_1 and the maximum moment stress resultant is Tg_1/s.

The calculations used to arrive at the numbers were carried out algebraically, so that each calculation leads to a formula and the influence of uncertainties and variations in material parameters can readily be assessed. Some modes apply to small areas: flexural collapse within a composite panel is an example, since it must occur within the compass of a frame if it is not to involve frame bending

Table 1

mode	sketch	m (N)	note
1		829	frame bolts yield; rotation about toes
2		945	hinge in panel
3		1244	frame bolts yield; rotation about heels
4		2571	hinge in angle leg
5		3344	frame and panel

too. Other modes only apply to large areas: for instance, failure by fracture of frame bolts obviously does not apply to areas which do not include the connection between adjacent frames.

The table shows that the bolted connections between the frames are weak by comparison with the frames themselves, whereas the bending moment capacity of the composite panels is about the same as the capacity of the bolted connections. This suggests that the capacity of the wall to resist pressure is limited *either* by the frame bolts *or* by the strength of the composite panels between the frames.

6. GLOBAL STRENGTH OF SEGMENTS OF FIREWALL

The wall can be thought of as a sequence of right-angled triangular segments, alternately base up and base down, each segment corresponding to one of the triangles of the N-form truss. The base of each triangle is bolted or welded to the ceiling or the floor, and the other two sides are clamped to a vertical or a diagonal of the truss.

The triangles are almost identical, although not precisely so, because the relation between the layout of the frames and the layout of the truss varies between segments. If we neglect that variation, each triangular segment can be treated as part of an infinite plate between parallel abutments, supported to form the infinite sequence of right-angled triangular segments sketched in Fig. 3. Under a uniform pressure loading extended over the whole plate, each segment will deform identically, and symmetry then imposes some conditions of the deformation. If $w(x,y)$ is the deflection of triangular segment 1 in Fig. 3, $w(-x,b-y)$ is the deflection of segment 0, $w(a-x,\ b-y)$ is the deflection of segment 2, and so on. Symmetry and continuity impose additional conditions on the derivatives on the boundaries: for instance, on the vertical boundary between segment 2 and segment 3

$$w_x(0,y) = -w_x(0,b-y) \tag{1}$$

and therefore the rotation is zero at midheight, and the mean rotation is zero on the boundary between segments 2 and 3. The same condition applies on the inclined boundaries between 1 and 2, between 3 and 4, and so on. Each triangular segment has no rotation on its horizontal side, and no

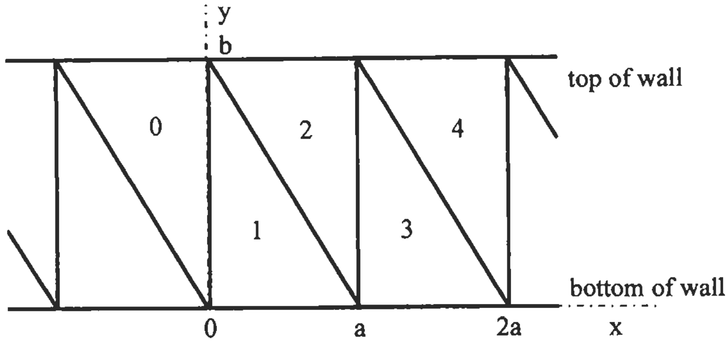

Fig. 3. Elevation and reference axes.

mean rotation on its inclined and vertical sides, and it is a good approximation to treat it as clamped on all three sides.

The next step is to relate the maximum moment stress resultant to the loading. Consider first a series of geometrically-similar plates, each characterized by an area A and loaded by a uniform pressure p, and made of the same material. It can then be shown by dimensional analysis that the maximum value of m must be proportional to pA. The form of the relationship is therefore:

$$m = pA/k \tag{2}$$

The value of k depends on the material properties, on the shape of the plate, and on how the edges are fixed.

We can calibrate this relationship by using analytic solutions for simple shapes. Values derived in this way are listed in Table 2. Each solution takes the plate edges as clamped. The table could be based on elastic solutions, for which the stress in the plate does not anywhere, reach the yield point, or it could be based on plastic solutions, which correspond to a condition in which the plate yields and a collapse mechanism develops. Since we wish to focus on the conditions that are present when the plate fails, the second plastic option is chosen. The values of k are derived from solutions to the problem of plastic collapse of a thin plate, within the well-established theoretical framework of plastic analysis of plates.

The analytic solution for a circular plate is exact. The other solutions are based on lower and upper bounds on collapse pressure, which can be derived from the lower and upper bound theorems of plasticity theory.

The table shows that the value of k does not depend strongly on the shape of the plate. This suggests that we can adopt a single value of k, and can use it to derive an approximate general relationship between pressure, area and maximum value of the moment stress resultant. The relationship ought to be applicable under the following conditions:

1. the plate is only supported at its edges, and not by internal supports;
2. the breadth and width of the plate are comparable, so that the plate is not long in one direction and narrow in the transverse direction: Table 2 suggests a maximum length/breadth ratio of 2;
3. the shape is convex.

Table 2

shape	$k=pA/m$	source	yield condition	notes
square	42.8	[6, 7]	Johansen	refined upper bound
square	32.0	[7, 8]	Johansen	lower bound
circle	35.4	[9]	Tresca	exact
2:1 rectangle	56.6	[8, 10]	Johansen	upper bound
hexagon	40.0	[11]	Johansen	lower bound
equilateral triangle	41.6	[6]	Johansen	upper bound

An approximate relationship between the area of a section of firewall and the maximum pressure it can sustain can be derived by bringing these results together. Taking the smallest value of 829 N from Table 1, and taking k as 50 from Table 2, the relationship is

$$p = 41450/A, \tag{3}$$

where p is in N/m^2 and A is in m^2. Most of the triangular segments have an area of aboout $14.55\,m^2$, the precise value depending on the detailed layout. The corresponding breakup pressure is therefore approximately $2.8\,kN/m^2$ (0.028 bars), much smaller than the calculated maximum pressure at point P1. This indicates that the firewall cannot withstand the pressure of the explosion in module C. A small number of triangular segments are slightly larger at $16.2\,m^2$, and have a correspondingly smaller breakup pressure.

7. DYNAMIC RESPONSE: ELASTIC MODEL

Section 6 gives us an estimate of breakup pressure under slow loading, in which the loading time is long by comparison with the lowest natural period of flexural oscillations. The next step is to consider the dynamic response of the firewall to the actual pressure pulse, which is quite short (between 100 and 200 ms), so that the dynamic response may be quite different from the response to the same maximum pressure applied slowly.

Two idealisations were used. The first idealisation treats the deflection of the firewall as elastic, but treats the critical deflection at which breakup begins as having both elastic and plastic components, since the bolts have some capacity to extend plastically before they break. The second more complete idealisation treats the wall as elastic-plastic, and is examined in Section 8.

The first step is to determine the natural frequency for a firewall segment, so that the loading time can be compared with the period corresponding to the lowest natural frequency. Appendix A is a summary of this calculation, which was carried out using the Rayleigh method.

The calculation idealises each firewall segment as a uniform plate with clamped edges. The mass is taken as uniformly distributed and equal to the average mass per unit area. A comparison "exact" calculation based on the actual distribution of mass in a typical segment confirms that this is an excellent approximation: the difference between the "exact" and "averaged" natural frequencies is 0.8%. The equivalent stiffness is more difficult to estimate, because the absence of structural continuity between adjacent frames leads to a significant contribution to the firewall flexibility from torsion in the angle sections between the frame corners and the nearest frame bolts. The equivalent plate flexural rigidity D was estimated as 10 000 N m. This was taken as the base case, but the study examined the sensitivity of the conclusions to the assumed value of D: this point is returned to later.

The estimated lowest frequency is 73 rad/s, which corresponds to a natural period of 86 ms. Looking back to Fig. 2, we can see that the loading time is of the same order as the natural period, neither much longer (so that the response would be quasi-static) nor much shorter (so that the response would correspond to impulsive loading).

The next step is to calculate the dynamic response. Pressure loading which is nearly uniform over a firewall segment primarily excites the lowest mode (corresponding to the lowest frequency). The lowest-mode response for central deflection can be written down as a formula which is a multiple of two terms. The first term is the deflection that would occur if the loading were applied slowly. The second term multiplies the first, and accounts for dynamic effects: it is a function of the natural frequency, the time that has elapsed since the pressure pulse began, and the duration and shape of the pulse. The multiplying second term is identical to the corresponding formula for a simple one-degree-of-freedom mass-on-spring system.

The results are shown in Fig. 4, which plots deflection at the centroid of a triangular segment against time; time is measured from the start of the triangular pulse in Fig. 2.

The deflection when the wall begins to break up can be estimated as the sum of two components:

1. the elastic deflection of a segment under the estimated collapse pressure under quasi-static loading, represented by x_Y in Fig. 5;
2. the additional deflection associated with plastic elongation of the frame bolts until they reach their specified minimum elongation, represented as $x_F - x_Y$ in Fig. 5.

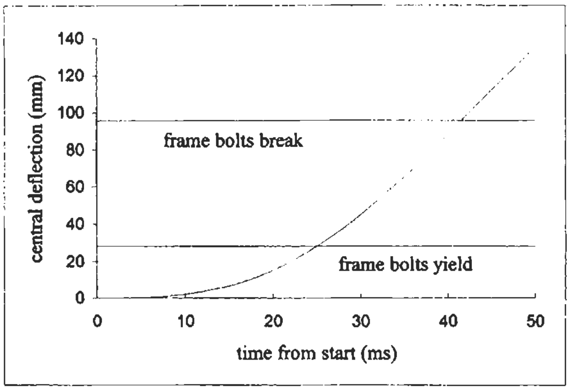

Fig. 4. Response calculated by elastic analysis.

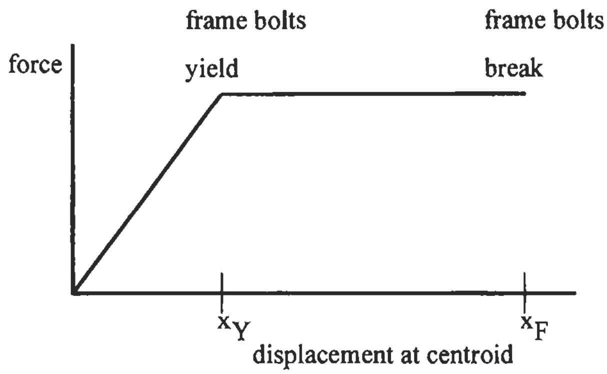

Fig. 5. Idealised relationship between applied force and displacement at centroid.

Taking D as 10 000 N m, the corresponding x_Y is 28 mm and $x_F - x_Y$ is 68 mm, so that the estimated deflection when frame bolts begin to break is 96 mm. This deflection is reached after 42 ms. The instantaneous pressure at that time is just below 0.1 bars, which is consistent with the value adopted for the onset of venting in the CFD calculation described in Section 2.

8. DYNAMIC RESPONSE: ELASTIC–PLASTIC MODEL

The analysis described in Section 7 treats the dynamic response as elastic, but determines the critical deflection at which the wall begins to break up as having both an elastic component (the general deflection of the firewall) and a plastic component (the additional deflection corresponding to plastic extension of the frame bolts). It can be improved by treating the dynamic response as elastic–plastic, explicitly taking into account the second phase of the motion, in which the wall is deflecting plastically by the plastic extension of frame bolts, but the frame bolts have not yet reached the extension at which they break.

The elastic–plastic analysis idealised the wall as a single degree-of-freedom mass-spring system. The function that relates the force applied to the firewall and the deflection x at the centroid of a triangular firewall segment is idealised in Fig. 5. The initial response is linear and elastic, up to the pressure at which the frame bolts yield: the corresponding deflection is denoted x_Y. The wall then deflects at constant force, until at a larger deflection x_F the most heavily-loaded frame bolts break. The pseudo-plastic deflection $x_F - x_Y$ corresponds to the extension of the frame bolts between yield and fracture. An equivalent mass factor takes account of the lower velocity of the edges than the sides.

The first part of the response is elastic up to first yield in the frame bolts: the solution is a relationship between displacement and time, and the initial conditions are zero displacement and zero velocity at the start of the pulse. The second part of the response is plastic: the solution is another relationship between displacement and time, with two integration constants determined by matching the solutions for the first and second parts of the response.

Figure 6 is the calculated relationship between wall segment centroid displacement and time, for the elastic-plastic model, and for five values of D.

Taking D as 10 000 N m, the breakup displacement is reached after 42 ms, which is close to the value calculated from the elastic analysis in Section 7. The physical reason for this is that the initial phase wall response is dominated by the effect of the pressure pulse on the mass of the wall, and the stiffness of the wall has only a secondary effect, at least in the first 50 ms or so. This can be confirmed by expanding the analytic solution as a power series in t, and noticing that the wall stiffness appears only in the smaller second term.

The time at which the frame bolts begin to break is insensitive to the assumed value of D, whose calculated value depends on how close the frame bolts are to the frame corners. Calculations in which D ranges from 10 000 Nm to 39 000 Nm show that the breakup time changes only from 42 s to 44 s after the start of the pulse, and so the assumed value of D has a negligible effect on the calculated pressure at breakup.

Once the first bolt has broken, the forces in neighbouring bolts rapidly increase, and they break soon afterwards. This adverse redistribution of internal forces leads to rapid separation of the firewall into panels. The pressure has still not reached its peak when the wall disintegrates into its component panels, and the remainder of the pressure pulse further accelerates the panels and projects them into module C.

9. ALTERNATIVE FAILURE MECHANISMS

The analysis described above takes the governing factor as tension failure of the frame bolts. Other modes of failure are possible. The composite panels could collapse as plates within the frames, but a calculation based on plate theory and a test on a 900 mm square panel shows that this requires a much higher pressure than does failure of frame bolts.

Another possible mode is tension failure of the clamps that hold the frames to the truss. Each clamp consists of two lengths of 3/8 in. studding, and can carry 37.9 kN. There are 42 clamps, and

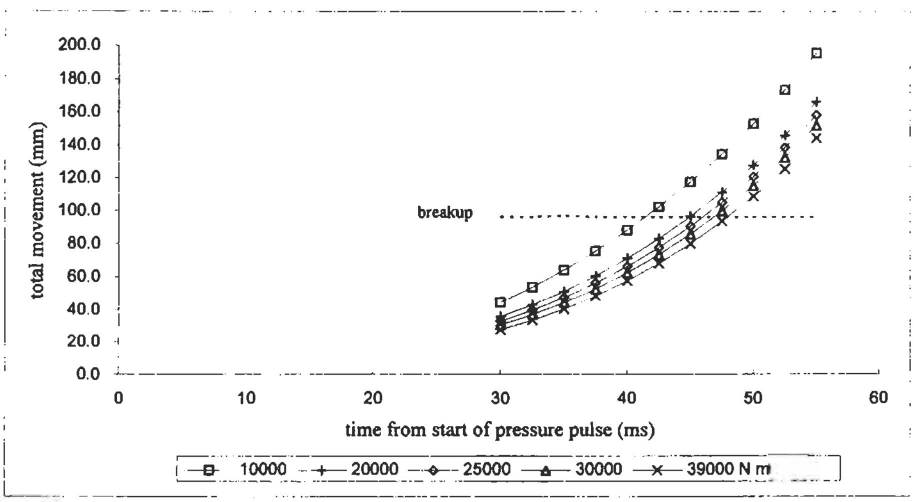

Fig. 6. Elastic–plastic analysis: movement at centroid as function of D.

together they can carry 1.59 MN. The total load that corresponds to the 19.5 kN/m^2 maximum pressure applied simultaneously across the whole firewall is 5.77 MN. The clamps are at midheight, and can be expected to carry at least half the total load. It follows that the clamps are not strong enough to carry the total load on the wall, and that the clamps would break if the the wall had not already broken up by failure of the frame bolts.

The analysis is based on plate theory, which is approximate because the deflection is not necessarily small by comparison with the effective thickness of the firewall. The effective thickness of the wall was estimated by finding the thickness which gives the same ratio between the fully-plastic membrane stress resultant at collapse in pure tension and the fully-plastic membrane stress resultant at collapse in pure bending [3], both calculated for the governing mode of frame-bolt failure in tension. The effective thickness turns out to be 80 mm for one direction of bending and 120 mm for the other. Moreover, the sides of the wall segments are not rigidly fixed at the top and bottom. It is known [4] that small inward movements at the edges of transversely-loaded plates much reduce the stiffening effect of membrane action, and an approximate calculation showed that in this instance an inward edge movement of the order of 1 mm would be enough to eliminate a significant increase in strength because of membrane effects. It was concluded that these effects could be neglected.

10. RESPONSE OF C/D AND A/B FIREWALLS

The wall between modules C and D was much stiffer and stronger than the wall between modules B and C. The estimated collapse pressure of one of its triangular panels under quasi-static slow loading is about 12 kN/m^2 (0.12 bars), compared to the peak pressure of 19.5 kN/m^2 at P1 in module C. The lowest natural frequency of one of its triangular segments is about 410 rad/s, corresponding to a period of 15 ms, and its response is not far from quasi-static.

The control room was in D module to the north of the C/D firewall, and had an additional wall of steel plate. Two survivors were in the control room at the time of the explosion. They were blown across the room, and saw that equipment near the wall had been damaged and that smoke was apparently entering at the top part of the wall. Accordingly, since the C/D wall is stronger than the B/C wall, it can be concluded independently that the B/C wall would have been more severely damaged by an explosion in C module than the C/D wall was.

The A/B wall was similar to the B/C wall in construction and arrangement. There is evidence from survivors that the A/B wall was not damaged. This supports the conclusion that the initial explosion was in C module. If the initial explosion had been in B module, it cannot be explained how the explosion leaves A/B intact but breaks down the stronger C/D wall. This is a particularly robust conclusion, and is of course independent of the calculations.

11. CONCLUSIONS

The analysis of the B/C firewall is consistent with the conclusion of the public inquiry, that an initial explosion in C module was followed by breakup of the firewall and projection of panel fragments into B module.

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APPENDIX

Estimate of lowest natural frequency of firewall segment

An upper bound to the lowest natural frequency w of a plate with all edges clamped, uniform mass per unit area m and uniform plate flexural rigidity D, can be estimated by Rayleigh's method from

$$\omega^2 = \frac{D}{\mu} \frac{\int_A \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 \mathrm{d}A}{\int_A w^2 \,\mathrm{d}A}$$

where $w(x, y)$ is a arbitrary deflection function which satisfies the kinematic boundary conditions, and both integrals are over the area of the plate. We consider a triangular plate whose vertices are (0, 0) (a, 0) and (0, b), and take

$$w(x, y) = \left(\frac{x}{a}\right)^2 \left(\frac{y}{b}\right)^2 \left(1 - \frac{x}{a} - \frac{y}{b}\right)^2 3^6 w_0$$

which satisfies the boundary conditions for a clamped plate. The calculation is assisted by the integral

$$\int_0^b dy \int_0^{a(1-y/b)} \left(\frac{x}{a}\right)^l \left(\frac{y}{b}\right)^m \left(1 - \frac{x}{a} - \frac{y}{b}\right)^n \mathrm{d}x = \frac{l!m!n!}{(l+m+n+2)!} ab$$

which is a special case of a standard integral quoted by Gradshteyn and Ryzhik [5]. After some algebra:

$$\omega^2 = 4004 \frac{D}{\mu a^2 b^2} \left(\left(\frac{a}{b}\right)^2 + 1 + \left(\frac{b}{a}\right)^2\right).$$