



A Latent Factor Model of European Exchange Rate Risk Premia

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The floating of a number of European currencies in 1992–1993 created a new body of data on foreign exchange risk premia, or deviations from uncovered interest rate parity (UIP). In this paper, excess returns to investments in SEK, NOK, FIM, GBP, ITL and ESP against the DEM are investigated. First, univariate GARCH-M models are estimated for each currency and UIP is tested. UIP fares as badly on this data set as in most other studies. Then a latent factor GARCH model that takes common effects in the different currency markets into account is applied. The risk premia are modelled as functions of time varying factor variances. A Kalman filter is used to identify the unobservable risk factors. A one-factor model that allows for idiosyncratic risk seem to fit the data quite well. However, the puzzling finding is made that the factor risk does not appear to be priced. Copyright © 1999 John Wiley & Sons, Ltd.

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SUMMARY

The floating of a number of European currencies in 1992–1993 created a new body of data on foreign exchange risk premia, or deviations from uncovered interest rate parity (UIP). Excess returns to investments in six European currencies relative to investments in German Marks are investigated. The currencies considered are Swedish Kronor, Norwegian Kronor, Finnish Marks, British Pounds, Italian Lire and Spanish Pesetas. Owing to the short time span considered, daily data on exchange rates and overnight interest rates are used.

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Using univariate tests, UIP fares badly on this data set as in most other studies. The currencies of high interest rate countries tend to appreciate instead of depreciate, as predicted by the theory. The only currencies for which uncovered interest rate parity cannot be rejected are Finnish Marks and British Pounds. In most of the empirical literature on foreign exchange rates premia, the Jensen inequality term is assumed to be small enough to be ignored. In the present tests of UIP, a Jensen inequality term is explicitly allowed for. However, it turns out not to be significantly different from zero.

Next, one possible explanation for the failure of UIP to hold is considered. The explanation is that the assumption of a constant risk premium is not valid. Therefore, a time varying premium is allowed for. It is assumed that a relatively small number of factors can account for the movements

in the risk premia of all the currencies, but that the factors are unobservable, i.e. a latent factor model that takes common effects into account is applied. A one-factor model that allows for idiosyncratic countries have common sources of risk. However, the factor risk does not appear to be priced.

INTRODUCTION

Uncovered interest rate parity (UIP) remains a key assumption in international macroeconomic modelling in spite of a massive body of empirical research rejecting such a relation between nominal interest rates and exchange rates. If UIP does not hold, there must be either systematic forecast errors or risk premia (or both) in foreign currency markets. While UIP is almost always rejected in empirical tests, specific alternative models of how the risk premium is determined have not found much empirical support either. McCallum (1994), Lewis (1995) and Engel (1996) provide surveys of this literature.

Genuine risk premia reward the investor for taking on (non-diversifiable) risk. According to standard asset pricing models, the risk premium is determined by the variance of asset returns, the covariance with the market portfolio, or the covariance with the marginal utility of consumption depending on what asset pricing model is used. In general equilibrium models, like that of Lucas (1982), the source of risk is the covariance between monetary shocks and output shocks.

Empirical studies using consumption data to test Euler equations have had difficulties explaining the behaviour of exchange rate risk premia (Hodrick, 1989; Kaminsky and Peruga, 1990; Backus *et al.*, 1993). Consumption does not vary enough to explain the variation in *ex post* returns unless consumers are implausibly risk averse. Macklem (1991) and Bekaert (1996) simulate versions of the Lucas (1982) general equilibrium model but are unable to mimic the behaviour of observed foreign exchange risk premia. In short, attempts to explain risk premia using data on the observable variables suggested by theoretical models have generally not been successful.

In this paper, risk premia on investments in SEK, NOK, FIM, GBP, ITL and ESP against the DEM are considered. First, univariate GARCH-M

models are estimated for each currency in order to test UIP, while allowing for a Jensen inequality term and a constant risk premium. The results are mixed. Next, a time varying risk premium is allowed for. An analysis of the residuals from the univariate models indicate that a multivariate specification would be appropriate, since the innovations to the conditional variances of the different currencies are correlated. Multivariate GARCH models are cumbersome since the number of parameters to be estimated is large. It is necessary to impose some structure on the covariances of the assets.

A latent factor GARCH model is used. In this framework, risk premia on the different assets covary because they are driven by common risk factors. However, the factors are not directly observable. A Kalman filter is used to extract the factors from the observable excess returns. The idea is that the sources of risk in the economy are not directly observable, but that risk premia are driven by movements in a limited number of risk factors. The modest questions that can be answered within this framework are whether common factors with time varying variances can be found in excess *ex post* returns and whether these returns are higher when the conditional variances are high. Hence, the existence of a specific type of risk premium can be confirmed or rejected but the sources of the risk are not identified in terms of observable variables.

In methodology, the paper is similar to Diebold and Nerlove (1989) and King *et al.* (1994). Diebold and Nerlove (1989) study the behaviour of nominal exchange rates using first a univariate GARCH model and then a latent factor model, applying a Kalman filter to identify the unobservable factor. However, they are not concerned with risk premia but rather in providing a good description of multivariate exchange rate movements. King *et al.* (1994) use a latent factor model with an ARCH structure to study the interaction among national stock markets.

The paper is organized as follows. In the next section, the data that are used are presented. In Section 3 simple tests of UIP are performed, using univariate GARCH-M models. Section 4 discusses a latent factor model of time varying risk premia. The risk premia are derived as functions of the conditional variances of the risk factors. In Section

5 the multivariate model is estimated using a Kalman filter to identify the unobservable factors. Section 6 concludes.

DATA

Since the countries in question have only had floating exchange rates for 4 years, we choose high frequency data to get as much information as possible on the time series behaviour of the data. Unless one finds a way to distinguish between term premia and exchange rate risk premia, it is necessary to use interest rates with non-stochastic returns, which rules out, for instance, holding period yields on government bonds. Daily data on overnight interest rates and exchange rates for Sweden (SEK), Norway (NOK), Finland (FIM), Spain (ESP), Italy (ITL), United Kingdom (GDP) and Germany (DEM) from 4 January 1993 to 9 April 1996 are obtained from the Sveriges Riksbank. This gives 800 observations, which is a relatively small sample for daily data. Hence, the results presented will have to be viewed as tentative. The *ex post* return to an investment in a

specific currency over the return to an investment in DEM is defined as $r_t^* - r_t + s_{t+1} - s_t$ or the foreign interest rate minus the German interest rate from t to $t+1$ plus the log of the exchange rate (the price of foreign currency in terms of DEM) at $t+1$ minus the log of the exchange rate at t .

Figure 1 shows *ex post* excess returns on SEK and ITL compared with DEM for the period 1 October 1994 to 30 June 1995, annualized yields. It can be seen that they are positive on average (0.05 means five percentage points). There also seem to be volatility clusters in the sense that high volatility is followed by high volatility and low volatility is followed by low volatility. The variances of the two series appear to be positively correlated. This suggests that a multivariate GARCH model may be appropriate.

Table 1 shows some descriptive statistics on the *ex post* returns. The means are not significantly different from zero. There is no evidence of autocorrelation as indicated by the heteroskedasticity consistent Box-Ljung statistics from Milhøj (1985) in the last column. As evident from the excess kurtosis and skewness measures, the excess returns are far from normally distributed.

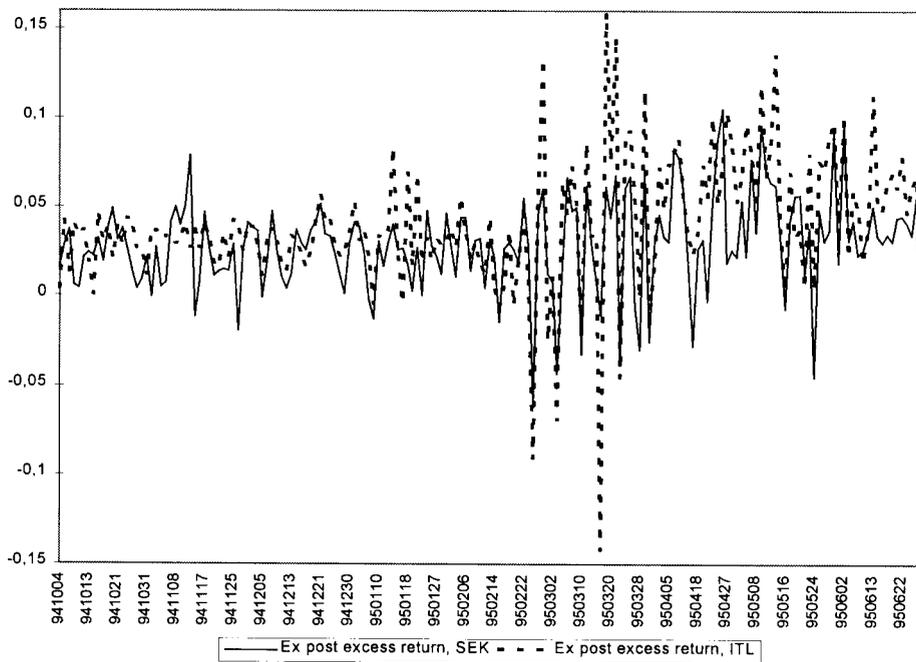


Figure 1. *Ex post* returns on SEK and ITL

Table 1. Statistics on *ex post* excess returns

	Mean	Variance	Excess kurtosis	Skewness	Box-Ljung
Sweden	4.347×10^{-5} (0.75)	4.087×10^{-5}	1.223 (0.00)	-0.286 (0.00)	10.267 (0.42)
Finland	6.553×10^{-5} (0.66)	1.796×10^{-5}	2.236 (0.00)	0.213 (0.00)	10.691 (0.38)
Norway	-1.042×10^{-5} (0.86)	0.298×10^{-5}	2.654 (0.00)	-0.308 (0.00)	16.134 (0.09)
UK	-13.35×10^{-5} (0.85)	4.087×10^{-5}	2.807 (0.00)	-0.298 (0.00)	13.155 (0.22)
Italy	5.221×10^{-5} (0.819)	4.167×10^{-5}	9.546 (0.00)	-1.096 (0.00)	8.542 (0.58)
Spain	-8.908×10^{-5} (0.56)	1.837×10^{-5}	10.631 (0.00)	-0.950 (0.00)	5.108 (0.88)

p values in parentheses.

TESTING UNCOVERED INTEREST RATE PARITY

We will consider investments in securities whose returns are risk-free in their respective currencies of denomination. Given general equilibrium and no arbitrage, a pricing kernel that prices the securities can be derived. Starting from the (nominal) pricing kernels of domestic and foreign assets, M_t and M_t^* , we have

$$E[M_t R_{t-1} | \Omega_{t-1}] = 1 \quad \text{and} \quad E[M_t^* R_{t-1}^* | \Omega_{t-1}] = 1 \quad (1)$$

where R_t is the domestic nominal interest rate and R_t^* is the foreign nominal interest rate. For instance, letting M_t denote the intertemporal marginal rate of substitution, (1) is the first-order conditions of the consumption capital asset pricing model. In order to be able to compare the returns on the two securities from the domestic investor's perspective, we can price the foreign asset in terms of the domestic currency,

$$E\left[M_t R_{t-1}^* \frac{S_t}{S_{t-1}} \middle| \Omega_{t-1}\right] = 1 \quad (2)$$

where S_t is the price of foreign currency in terms of domestic currency. We let lowercase letters denote logarithms and assume that all variables are lognormally distributed. Using Equation (1) and taking logs of (2) we obtain the expression

$$\begin{aligned} & E[r_{t-1}^* - r_{t-1} + s_t - s_{t-1} | \Omega_{t-1}] \\ &= -\frac{1}{2} \text{var}(s_t | \Omega_{t-1}) - \text{cov}(m_t, s_t | \Omega_{t-1}) \end{aligned} \quad (3)$$

If covered interest rate parity holds, the left-hand-side will equal the difference between the forward foreign exchange rate and the expected future spot exchange rate. This difference would be driven to zero by arbitrage activities if agents were risk neutral. Hence, the expression in (3) is sometimes referred to as a foreign exchange *risk premium*. In a consumption-based model, where agents are risk averse and have constant relative risk aversion, the last term on the right-hand-side could (ignoring inflation) be rewritten as $-\gamma \text{cov}(c_t, s_t | \Omega_{t-1})$, where c_t is the rate of growth of consumption and γ is the coefficient of relative risk aversion. A risky asset with a positive risk premium has low returns in times of high marginal utility (low consumption). With risk-neutral agents ($\gamma = 0$), the covariance term would disappear from the right-hand-side of expression (3). Thus, in a sense the covariance term could be thought of as the 'true' risk premium.

The first term on the right-hand-side in expression (3) stems from Jensen's inequality. It is difficult to give it an economically meaningful interpretation. If the home country and the foreign country are reversed, increased variance of the exchange rate will have the opposite effect on the risk premium. A number of papers have studied the size of the Jensen inequality term and found it to be negligible (Engel, 1984; Cumby, 1988; Hodrick, 1989; Backus *et al.*, 1993). In his survey of the literature, Engel (1996) concludes 'In the empirical literature on foreign exchange risk premia, the Jensen inequality terms can be ignored because of their small size'. In our test of UIP we will allow

for (and test the significance of) the Jensen inequality term.

Assuming rational expectations with respect to the information set $Z_t \subset \Omega_t$, we can derive the following expression from Equation (3),

$$s_t - s_{t-1} = r_{t-1} - r_{t-1}^* - \frac{1}{2} \text{var}(s_t | Z_{t-1}) + \text{cov}(-m_{t^*}, s_t | Z_{t-1}) + u_t \tag{4}$$

where $E[u_t | Z_{t-1}] = 0$. Assuming that there exists a pricing kernel such that the covariance term in expression (4) is equal to a constant C for all t , we would get

$$s_t - s_{t-1} = C + r_{t-1} - r_{t-1}^* - \frac{1}{2} h_t + u_t \tag{5}$$

where $h_t = E_t[u_t^2]$. Expression (5) can be estimated for each country with the following GARCH(p, q)-M model¹

$$s_t - s_{t-1} = b_0 + b_1(r_{t-1} - r_{t-1}^*) + b_2 h_t + u_t, \tag{6a}$$

$$u_t \sim n(0, h_t) \tag{6b}$$

$$h_t = a_0 + \sum_{i=1}^p a_i u_{t-i}^2 + \sum_{j=1}^q c_j h_{t-j} \tag{6b}$$

where h_t is the conditional variance of the change in the exchange rate. The coefficient b_0 is a constant risk premium. The coefficient b_1 should equal one if uncovered interest parity holds. The coefficient b_2 captures the effect of the Jensen inequality term and should be equal to minus one half according to the model.

We estimated the three specifications that the Engle LM tests suggested were most likely to match the daily returns data. The best of these specifications for the respective series are reported in Table 2 (the three candidates suggested by the LM tests were GARCH(1, 1)-M, GARCH(2, 1)-M and ARCH(5)-M). The estimated models are well behaved in the sense that the standardized residu-

Table 2. Parameter estimates of the GARCH(p, q)-M model in (6)

	Sweden	Finland	Norway	UK	Italy	Spain
b_0	-0.0013 (-1.532)	9.95×10^{-5} (0.692)	2.46×10^{-5} (0.285)	-0.0021 (-1.272)	-0.0013 (-2.816)	-0.0014 (-7.128)
b_1	-10.092 (-1.864)	-4.581 (-0.974)	-5.784 (-3.278)	-6.063 (-0.710)	-12.469 (-2.989)	-16.048 (-9.978)
b_2	16.609 (0.743)	-0.005 (-0.001)	-16.426 (-0.483)	12.814 (1.030)	0.031 (0.104)	0.007 (0.029)
a_0	2.4×10^{-5} (9.589)	4.88×10^{-3} (6.146)	0.11×10^{-5} (9.447)	9.13×10^{-5} (9.463)	0.77×10^{-5} (7.670)	0.15×10^{-5} (5.074)
a_1	0.060 (2.001)	0.200 (3.990)	0.265 (4.540)	0.053 (1.270)	0.167 (2.736)	0.632 (9.937)
a_2	0.124 (2.219)		0.065 (1.686)	0.063 (1.541)	0.204 (4.121)	
a_3	0.120 (2.444)		0.161 (3.953)	0.113 (2.356)		
a_4	0.036 (1.324)		0.067 (1.778)	0.059 (2.340)		
a_5	0.063 (1.957)		0.104 (2.692)	0.086 (1.951)		
c_1		0.426 (6.289)			0.475 (27.688)	0.509 (21.554)
$F(b_1 = 1)$	4.198 (0.040)	1.409 (0.235)	14.778 (0.000)	0.639 (0.408)	10.424 (0.001)	112.362 (0.000)
$F(b_0 = 0, b_1 = 1, b_2 = 0.5)$	5.432 (0.143)	2.632 (0.452)	15.464 (0.001)	2.110 (0.550)	13.556 (0.003)	134.142 (0.000)

t values using conventional standard errors in parentheses.
 p values for the F -tests in parentheses.

als and squared standardized residuals are not autocorrelated according to the Box-Ljung test statistics (with ten degrees of freedom).

The estimates of b_1 are all negative and larger than what is usually found when testing this specification of UIP. Three of the parameters are significantly different from zero. In four cases, the hypotheses that b_1 equals one is rejected. UIP is not rejected for Finland and the UK. Furthermore, the joint hypothesis that b_1 equals one and b_0 equals zero is rejected in three cases. The parameter b_2 measures the effect of the Jensen's inequality term. Although three of the estimates are large, none of them is significantly different from zero. Hence, Jensen's inequality does not appear to explain deviations from UIP in this data set.

A MODEL OF THE RISK PREMIA

In the previous section, the exchange rate risk premia were assumed to be constant over time while time varying variances were allowed to affect returns. Each country was also analysed in isolation, whereby cross effects between the currency markets were ignored. We now turn to a more general asset pricing model that takes (i) time varying risk premia, and (ii) common effects to all the Deutsche Mark exchange rates into account.

If the general theoretical model in (3) is combined with specific assumptions about the processes followed by excess returns and the pricing kernel, a convenient solution for the risk premium can be derived. Deviations from UIP are assumed to follow a K -factor linear model,

$$y_{it} = \mu_{it} + \sum_{k=1}^K \beta_{ikt} f_{kt} + v_{it} \quad (7)$$

where we will refer to y_{it} as the excess return to currency i , $y_{it} \equiv r_{i,t} - r_{t-1} + s_t - s_{t-1}$, μ_{it} is the conditional mean, f_{kt} is the k th factor and β_{ikt} is the corresponding factor loadings of currency i . v_{it} is the residuals or idiosyncratic risk, assumed to be white noise with variance σ_i^2 .² Furthermore, the factors are assumed to be uncorrelated with each other and with the residuals and have zero expected value, i.e. $E_{t-1}[f_{kt}] = 0$.

The K factors represent risks that are common to all the assets, while the residuals represent diversi-

fiable asset specific risk. For instance, one factor could be productivity disturbances to the German economy that effects all the Deutsche Mark exchange rates and another could be changes in the prospects for a European Monetary Union. From a Lucas (1982) type of model, we would expect a productivity shock and a monetary policy shock in each country. However, the different monetary policies are unlikely to be linearly independent of each other or of the real shocks. The number of orthogonal risk factors in the data is an empirical question. While the realization of a particular type of common shock in period t is unpredictable given the information in period $t-1$, the variance of the shock is not as it follows a GARCH process. As we will see, this is what drives the risk premia or the conditional means μ_{it} .

We assume that the factor loadings β_{ikt} are constants β_{ik} and that the factors are characterized by time varying variances λ_{kt} . An alternative assumption of constant factor variances but time varying factor loadings would lead to the same conditional variance of excess returns, namely

$$h_{it} = E_{t-1}[(y_{it} - \mu_{it})(y_{it} - \mu_{it})] = \sum_{k=1}^K \beta_{ik} \beta_{kt} \lambda_{kt} + \sigma_i^2 \quad (8)$$

Hence, the conditional variances of the excess returns are linear functions of the conditional factor variances, λ_{kt} . The pricing kernel is assumed to obey a K -factor linear process,

$$m_t = v + \sum_{k=1}^K \tau_k f_{kt} + w_t \quad (9)$$

The interpretation of (9) is analogous to that of (7). The following moment conditions are assumed to hold: $E_{t-1}[w_t] = 0$, $E_{t-1}[w_t v_t] = 0$, $E_{t-1}[w_t | f_{1t}, \dots, f_{Kt}] = 0$ and $E_{t-1}[w_t^2] = \sigma_w^2$. Hence, the mean zero innovations in (7) and (9) are assumed to be uncorrelated. The other moment conditions are equivalent to those presented earlier for (7).

From expressions (3), (7) and (9), ignoring the Jensen inequality term (which was found to be insignificant in the previous section), the expected returns (or conditional means) can be derived as functions of the conditional variances of the factors λ_{kt} , the factor loadings β_{ik} , and τ_k that will be interpreted as the price of k -factor risk. The expected return for currency i can be written as

$$\mu_{it} = \sum_{k=1}^K \beta_{ik} \lambda_{kt} \tau_k \quad (10)$$

The factor loading β_{ik} captures the sensitivity of currency i to volatility in the k th factor, λ_{kt} , while τ_k reflects the sensitivity of the pricing kernel that prices the risk.

ESTIMATES OF THE LATENT FACTOR GARCH MODEL

The idea behind the latent factor model is that excess returns are driven by movements in a limited number of risk factors of the economy. The main problem when estimating such models is that these factors are unobservable. There is a variety of methods for identifying the factors in (7). In Engle *et al.* (1990), Engle and Ng (1993) and Sellin (1996), the factors are approximated by factor representing portfolios of the included assets. Korajczyk and Viallet (1992) use a principal components method that is most useful when the number of assets in the model is very large. Here, as in Diebold and Nerlove (1989), a Kalman filter is used to capture the movements of the unobservable factor(s) from the observable *ex post* returns of the assets. This technique is also used in King *et al.* (1994), where in addition some of the factors are connected to macro variables and are thus 'partially observable'. Such a connection will not be possible to make in the present paper since we use daily data.

Since the number of parameters to be estimated increases quickly with the number of factors included in the model, the number of factors has to be kept small. In several papers, only one factor is assumed to exist and the potential presence of additional factors is not investigated. Korajczyk and Viallet choose to use five factors but find that their main results are robust to changes of the number. A likelihood ratio test can be used to determine whether an additional factor is needed or not. Hence, the model is first estimated with one factor. Additional factors are then included as long as they are significant.

Using the Kalman filter, the K unobservable factors and their conditional variances can be extracted from the observable excess returns. The processes followed by the observable and unobservable variables have to be specified, including

the way in which the observable variables are related to the unobservable. The Kalman algorithm then constructs an optimal guess of the unobservable variables in each period, using the observable variables and the parameters of the dynamic specifications. These parameters and the constructed time series of unobservable variables in turn imply values of the observable variables that can be compared with the actual values. Hence, each set of parameters is associated with a sequence of prediction errors. Maximum likelihood estimation is used to pick the set of parameters and the corresponding sequence of unobservable variables that minimise the prediction errors.

The state space model of the Kalman filter is particularly simple in this context since the unobservable risk factors follow a process without serial correlation (by assumption). Combining (7) and (10), the observation equation that links the factors to the observable returns can be written as

$$y_{it} = \sum_{k=1}^K \beta_{ik} \lambda_{kt} \tau_k + \sum_{k=1}^K \beta_{ik} f_{kt} + v_{it}, \quad i = 1, \dots, N \quad (11)$$

where the y_{it} are the observed returns. The first term on the right-hand-side of (11) is the risk premium. It consists of the conditional variances of the factors λ_{kt} multiplied by the constant prices of k -factor risk τ_k and the sensitivities of asset i to the factors β_{ik} . The second term is the same factor sensitivities β_{ik} multiplied by the innovations to the factors. The expected value of these innovations is zero. The factors are assumed to be uncorrelated with each other. Finally, v_{it} are the asset specific innovations, assumed to be uncorrelated with each other, over time and to have zero expected value.

The covariance matrix of excess returns H_t is $\beta \Lambda_t \beta^T + \Omega$, where Λ_t is the $K \times K$ diagonal matrix of conditional variances of the factors, β is the $N \times K$ matrix of factor loadings and Ω is the $N \times N$ time invariant variance-covariance matrix of the asset specific shocks. It is assumed to be diagonal, i.e. all correlation between asset returns stems from their joint dependence on the unobservable factors. An informal indication of whether the common effects have been removed from asset returns can be obtained from the correlations of the asset specific disturbances given the estimated factors. They will be zero if the factors as modelled actually capture the common effects.

Table 3. Estimates from the one-factor model

GARCH parameters	Ψ_1	Φ_1				
	0.135 (4.63)	0.837 (22.97)				
	Sweden	Finland	Norway	UK	Italy	Spain
Factor loadings	β_1 4.51×10^{-3} (8.14)	β_2 2.57×10^{-3} (7.79)	β_3 1.43×10^{-3} (8.26)	β_4 5.76×10^{-3} (7.38)	β_5 5.89×10^{-3} (8.22)	β_6 2.71×10^{-3} (8.10)
Standard deviations of country specific shocks	$\Omega_{11}^{1/2}$ 4.33×10^{-3} (41.36)	$\Omega_{22}^{1/2}$ 3.37×10^{-3} (39.07)	$\Omega_{33}^{1/2}$ 0.94×10^{-3} (27.42)	$\Omega_{44}^{1/2}$ 10.48×10^{-3} (50.99)	$\Omega_{55}^{1/2}$ 4.13×10^{-3} (62.91)	$\Omega_{66}^{1/2}$ 3.34×10^{-3} (42.62)
Price of factor risk	τ 0.010 (0.28)					
Function value	19 321					
Q(10)	17.01 (0.07)	25.66 (0.004)	29.58 (0.001)	21.73 (0.02)	30.53 (0.001)	12.49 (0.25)
QS(10)	177.7 (0.00)	212.1 (0.00)	103.3 (0.00)	92.8 (0.00)	31.9 (0.00)	44.7 (0.00)

t values using conventional standard errors in parentheses.
p values for specification tests in parentheses.

The conditional variances of the factors are assumed to follow GARCH(*p*, *q*) processes,

$$\lambda_{kt} = \Psi_{k0} + \sum_{i=1}^p \Psi_{ki} E[f_{k,t-1}^2 | I_{t-1}] + \sum_{i=1}^q \Phi_{ki} \lambda_{k,t-i}$$

$$k = 1, \dots, K \tag{12}$$

The conditional variances of the different factors are assumed to be independent of each other. The unconditional variance of factor *k* is given by

$$V[f_{kt}] = \Psi_{k0} \left[1 - \sum_{i=1}^p \Psi_{ki} - \sum_{j=1}^q \Phi_{kj} \right]^{-1} \tag{13}$$

Since the scaling of the factors is irrelevant, we can normalize the unconditional variance to one by setting $\Psi_{k0} = 1 - \sum_{i=1}^p \Psi_{ki} - \sum_{j=1}^q \Phi_{kj}$. Following King *et al.* (1994), the updating recursions for the Kalman filter in this case can be derived as

$$f_{it|t} = E[f_i | I_t] = \Lambda_t \beta^T (\beta \Lambda_t \beta^T + \Omega)^{-1} (y_t - \beta \Lambda_t \tau) \tag{14}$$

for the factors and

$$\Lambda_{t|t} = V[f_i | I_t] = \Lambda_t - \Lambda_t \beta^T (\beta \Lambda_t \beta^T + \Omega)^{-1} \beta \Lambda_t \tag{15}$$

for the conditional variances of the factors. When combining the updating equations with (12), we use the fact that $E[f_{kt}^2 | I_t] = E^2[f_{kt} | I_t] + V[f_{kt} | I_t]$.

The likelihood function to be maximized is

$$\ln L = C - \frac{1}{2} \sum_{t=1}^T (\ln |H_t| + (y_t - \beta \Lambda_t \tau)^T H^{-1} (y_t - \beta \Lambda_t \tau)) \tag{16}$$

The Kalman filter identifies the factors and their conditional variances. The GARCH parameters Ψ_i and $\Phi_{i\tau}$, the variances of the asset specific shocks $\Omega_{i\tau}$, the elements of the β matrix of factor loadings or the sensitivities of the assets to movements in the factor and τ , the prices of factor risks, are estimated by maximum likelihood. The parameter estimates for the one-factor model are shown in Table 3. A GARCH(1, 1) turned out to be sufficient as higher-order terms were insignificant and did not result in significant increases of the likelihood function.

The factor loadings that relate the excess returns to the conditional variance of the factor are all positive as expected and highly significant. The Italian lira is most sensitive to movements in the factor and the Norwegian kronor the least. The factor loadings are small in absolute value but this does not necessarily mean that the risk premia are small, since they depend on the product of the

Table 4. Estimates from the one-factor model with time varying idiosyncratic risk

GARCH parameters	Ψ_1	Φ_1						
	0.149 (7.12)	0.840 (34.92)	Sweden	Finland	Norway	UK	Italy	Spain
Factor loadings	β_1 5.92×10^{-3} (21.50)	β_2 3.33×10^{-3} (16.73)	β_3 1.91×10^{-3} (26.63)	β_4 7.94×10^{-3} (18.97)	β_5 6.13×10^{-3} (21.10)	β_6 $3.23.71 \times 10^{-3}$ (20.89)		
Conditional variance of country specific shocks	ϕ_{10}	ϕ_{20}	ϕ_{30}	ϕ_{40}	ϕ_{50}	ϕ_{60}		
	2.1×10^{-6} (3.54)	3.0×10^{-6} (5.73)	0.9×10^{-6} (14.08)	1.1×10^{-4} (20.31)	0.7×10^{-6} (3.36)	0.1×10^{-6} (2.91)		
Price of factor risk	ϕ_{11} 0.172 (5.57)	ϕ_{21} 0.282 (8.36)			ϕ_{51} 0.120 (3.86)	ϕ_{61} 0.121 (6.83)		
	ϕ_{12} 0.750 (16.30)	ϕ_{22} 0.478 (11.55)			ϕ_{52} 0.861 (28.45)	ϕ_{62} 0.900 (78.95)		
	τ 0.029 (0.56)							
Function value	19 585							
Q(10)	14.83 (0.14)	16.69 (0.08)	15.04 (0.13)	22.40 (0.01)	7.50 (0.68)	6.83 (0.74)		
QS(10)	5.51 (0.85)	12.03 (0.28)	32.98 (0.00)	86.23 (0.00)	0.63 (1.00)	2.65 (0.99)		

t values using conventional standard errors in parentheses.
p values for specification tests in parentheses.

factor loadings, the price of factor risk and the conditional variance of the factor. The unconditional variance of the factor is normalized to 1.00, which makes it large compared with daily returns. Hence, we expect factor loadings to be small. The GARCH effects in the factor variance are also highly significant. The only insignificant parameter is τ , which can be interpreted as the price of factor risk. Hence, although there is evidence of a common factor it does not appear to be priced.

The Box-Ljung test of the standardized residuals in Table 3 are not satisfactory. There is a substantial amount of autocorrelation in the residuals. The Box-Ljung test of the squared residuals, as well as Engle's test for ARCH (not reported), indicate that there are still ARCH effects that the model is unable to explain. These problems could point to the need for one more risk factor in the model. We attempted to estimate two-factor models.³ None of these models converged and there were strong indications during the iterations that the models

were overparameterized. Our interest then turned in a different direction.

Note that idiosyncratic country shocks seem to be very important in Table 3. This indicates that it could be worth the effort to model this type of risk more carefully. Hence, we next try a one factor model with time varying idiosyncratic risk. More specifically, we let the conditional variances of the country specific shocks vary over time as a GARCH(1, 1) process,

$$\Omega_{it} = \phi_{i0} + \phi_{i1}E[v_{i,t-1}^2|I_{t-1}] + \phi_{i2}\Omega_{i,t-1} \tag{17}$$

The resulting parameter estimates are reported in Table 4. Our attempts to model the conditional variances for Norway and the UK were not successful, so we let these be constant over time. However, the GARCH modelling of the other four countries' idiosyncratic risk proved successful and results in a much better specified model. The Box-Ljung statistics for the levels look fine except for the UK, while the Box-Ljung for the squares are

OK for the series which we managed to model with time varying idiosyncratic risk. The value of the likelihood function is much higher than for the model with constant idiosyncratic risk. The precision in the estimates of the factor loadings are better than in the previous model but the factor risk is still not priced.

CONCLUSIONS

Ex post returns to investments in SEK, NOK, FIM, GBP, ITL and ESP against the DEM from 01/01/1993 to 09/04/1996 display a number of interesting features. UIP fares as badly on this data set as in most other studies. The currencies of high interest rate countries tend to appreciate instead of depreciate, as predicted by the theory.

Our attempts to model the risk premia as functions of time varying second moments met with limited success. In the univariate regressions, the Jensen inequality term does not have a significant effect on the deviations from uncovered interest rate parity. We then estimate a latent factor model which allows for a multivariate specification that keeps the number of parameters to be estimated reasonably low. The risk premia are assumed to be driven by movements in one or more common sources of risk that are not directly observable. We estimate the unobservable factors from the observable asset returns using a Kalman filter.

The one factor model with time varying idiosyncratic risk fair reasonably well. The factor loadings are significant, indicating that there are common sources of risk. However, price of factor risk is not significant, implying that the model captures systematic risk but no systematic risk premia. We conclude that while a latent factor GARCH model describes movements in *ex post* returns quite well, it fails to explain the occurrence of exchange rate risk premia.

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NOTES

1. See Bollerslev *et al.* (1992, 1994) or Bera and Higgins (1993) for a survey of the literature on these types of time series models. The GARCH-M model is discussed by Engle *et al.* (1987).
2. It is not necessary to assume constant idiosyncratic risks. King *et al.* (1994) apply a GARCH model also for asset specific risks and are thereby able to test whether the price of this risk is zero as expected from the theory.
3. Birger Nilsson generously provided us with his RATS code for estimating a multifactor model.

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